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# A GAUSSIAN SUM BAYESIAN APPROACH TO PASSIVE BEARINGS ONLY TRACKING*



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## 1. INTRODUCTION

Among anti-submarine warfare (ASW) target motion analysis (TMA) algorithms for mobile platforms, passive bearings only tracking is the most basic segment. The present state-of-the-art of tracking by this method is not satisfactory [1]. None of the existing extended Kalman filter algorithms yield both accurate tracks and an accurate solution quality indicator. Generally, these algorithms perform no better than much simpler manually controlled algorithms. Several approaches to the use of the recursive filtering theory are discussed in references [1, 2]. The approaches can all be referenced to a basic text book Extended Kalman Filters. Differences among the algorithms correspond to choice of coordinate systems, whether one aligns the linearization point of the nonlinear measurement to the line of sight or not, whether or not one limits artificial range collapse, and whether or not one uses some other form of divergence control. The difficulties are quite well documented in reference [1].

The severity of the undesired behavior differs from algorithm to algorithm. This difference depends on the nature of the algorithm and the accuracy of the measurements, the rate of the measurements, the range and data rate, and the geometric configuration of OWN ship and target ship. However, the fact that none of the algorithms generally give satisfactory performance leaves one to question whether or not the difficulties lie with the nature of the extended Kalman filter which is being applied to this problem. It is the purpose of this paper

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to point out the difference in performance between a simple extended Kalman filter and an optimal Bayesian filter in pictorial form. And, to show reasons why extended Kalman filters do not perform satisfactorily.

The actual implementation of the Bayesian optimal filter is a Gaussian sum approach documented in references [2, 4, 5]. However, the purpose of this paper is not to demonstrate the utility of the Gaussian sum approach in nonlinear filtering, but instead to show the difference between an optimal Bayesian filter implemented in any manner whatsoever and a linearized or extended Kalman filter. It is hoped that the following description will give the reader insight into the nature of both the optimal Bayesian filter and the approximation nature inherent in any extended Kalman filter.

## 2. PASSIVE BEARINGS ONLY TRACKING EXAMPLE

A simple two-dimensional version of the bearings only tracking problem is considered. The state vector is chosen to be two-dimensional (latitude, longitude) for simplicity. The extention to the case of four state variables, including bearings and speed information is straight forward. Again, for simplicity, OWN ship is assumed to be traveling at a constant speed around a circular path. Any trajectory for OWN ships motion could have been used.

These simplifications have been introduced because the purpose of this paper is to illustrate the nature of the optimal Bayesian approach to this problem and the effects of various linearization approximations, not to solve the problem in the most expeditious manner.

Once an engineer sees the nature of an "optimal" solution to a nonlinear filtering problem, he can often develop a simpler, but adequate suboptimal (often a linearization type) filter.

The state vector propagates according to the linear plant model.

$$\underline{x}_{k+1} = \underline{x}_k + \underline{w}_k \qquad 1$$

where the two-dimensional state represents latitude and longitude coordinates.

$$\underline{x}_k = \begin{pmatrix} x_k \\ y_k \end{pmatrix}$$

and the state is observed by the scalar nonlinear measurement function:

$$z_k = h_k(\underline{x}_k) + v_k \qquad 2$$

where:

$$h_k(\underline{x}_k) = \tan^{-1}\left[(y_k - \sin\beta_k)/(x_k - \cos\beta_k)\right] \qquad 3$$

$$\beta_k = \beta_0 + \dot{\beta}(k-1) \qquad 4$$

where $\beta_0$ and $\dot{\beta}$ are given constants. The statistics of the a priori random variables $\underline{x}_0$, $v_k$, and $\underline{w}_k$ which are white independent Gaussian random variables are:

$$E(v_k) = 0 \qquad E(\underline{w}_k) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad E(\underline{x}_0) = \hat{\underline{x}}_0 \qquad 5$$

$$E(v_k\, v_j) = \sigma_v^2\, \delta_{kj} \qquad E(\underline{w}_k\, \underline{w}_j^T) = \begin{pmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{pmatrix} \delta_{kj} \qquad 6$$

$$E\left[(\underline{x}_k - \hat{\underline{x}}_k)(\underline{x}_k - \hat{\underline{x}}_k)^T\right] = P_0' \qquad 7$$

The above model arises in connection with the tracking geometry of Figure 1, where the target T at the position defined as $\underline{x}_k^T = (x_k, y_k)$ is undergoing a random walk in the two dimensional state space. The observer S is passively measuring the line of sight $\alpha$ as it travels in

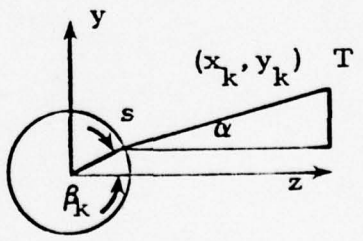



Figure 1. Geometrical Definition of Vector Tracking Example

a deterministic orbit around the unit circle. It is important to note that when $\dot{\beta}$ is equal to zero the target is unobservable and the filtering problem becomes degenerate.

The problem to be considered here is how to obtain the best possible estimate of the state $\underline{x}_k$ of the system (1-2) based on all data up to time $k$, $z^k$.*

*The collection of data up to time $k$ $\{z_1, z_2, \ldots z_k\}$ is defined as $z^k$.

## 3. BAYESIAN RECURSION RELATIONS

Since the state vector $\underline{x}_k$ is a random variable, in the probabilistic context of this discussion, the a posteriori density $p(\underline{x}_k | z^k)$ provides the most complete description possible of $\underline{x}_k$. This density is determined recursively from the Bayesian recursion relations given below. The filtering density is given first and the prediction density is next.

$$p(x_k | z^k) = c_k p(x_k | z^{k-1}) p(z_k | x_k) , \tag{8}$$

$$p(x_k | z^{k-1}) = \int_{R^n} p(x_{k-1} | z^{k-1}) p(x_k | x_{k-1}) dx_{k-1} . \tag{9}$$

where the normalizing constant $c_k$ is given by

$$1/c_k \triangleq p(z_k | z^{k-1}) = \int_{R^n} p(x_k | z^{k-1}) p(z_k | x_k) dx_k , \tag{10}$$

and the initial condition for (8) is

$$p(x_o | z^{-1}) \triangleq p(x_o) = N(y_o - \hat{x}'_o, P'_o) \tag{11}$$

The densities $p(z_k | x_k)$ and $p(x_k | x_{k-1})$ are determined from (1) and (2), and the a priori densities $p(v_k)$ and $p(w_{k-1})$.

$$p(z_k | x_k) = N(z_k - h_k(x_k), R_k) \tag{12*}$$

$$p(x_k | x_{k-1}) = N(x_k - x_{k-1}, Q_k) \tag{13*}$$

---

* $$N(x-\hat{x}, A) \triangleq \frac{1}{|A|^{1/2} \pi^{\eta/2}} \exp\left\{ -.5 (x-\hat{x})^t A^{-1} (x-\hat{x}) \right\}$$

## 4. THE EXTENDED KALMAN FILTER

The nonlinear filtering problem is solved when the density $p(x_k | z_k)$ can be obtained for all $k$. However, except when equation 2 is linear and the a priori distributions are Gaussian, it is generally impossible to determine $p(x_k | z^k)$ in a closed form using (8)-(11). In the linear Gaussian case, the relations describing the conditional mean and covariance of $p(x_k | z^k)$ are the well-known Kalman filter equations (6). The difficulties associated with the explicit determination of the a posteriori density have led to the development of approximate procedures for estimating the state of nonlinear stochastic systems. The most commonly used approximation involves the assumptions that the a priori distributions are Gaussian and that the nonlinear system can be linearized relative to latest estimate yielding the "extended" Kalman filter which has seen widespread application to nonlinear systems.

The extended Kalman filter has performed well in many applications but there are numerous examples in which unsatisfactory results have been obtained. This has spurred the development of other procedures. Most of these either implicitly or explicitly, retain the assumption that $p(x_k | z^k)$ is Gaussian and essentially provide a means for modifying the mean and covariance of the density. However, the Gaussian assumption greatly reduces the generality or richness of possible a posteriori densities by throwing out all possibility of more realistic multimodal densities.

Starting at stage 0 and linearizing about a point $\bar{x}_k$ in state space and dropping all but the first order terms, we can solve the Bayesian recursion relations to find:

$$\Omega_k = H(\bar{x}_k) P'_k H^T(\bar{x}_k) + R_k \qquad 14$$

$$K_k = P'_k H^T(\bar{x}_k)\, \Omega_k^{-1} \qquad 15$$

$$\hat{x}_k = \hat{x}'_k + K_k(z_k - h(\hat{x}'_k)) \qquad 16$$

$$P_k = (I - K_k H(\bar{x}_k))\, P'_k \qquad 17$$

$$P'_{k+1} = P_k + Q_k \qquad 18$$

$$\hat{x}'_{k+1} = \hat{x}_k \qquad 19$$

The various versions of the extended Kalman filter are simply different choices of $\bar{x}_k$. The most common text book choice is $\hat{x}'_k$,

$$H(\bar{x}_k) = \partial h(x)/\partial x \,|_{\bar{x}_k} .$$

Iterated extended Kalman filters require processing the data through equations (14), (15), and (16) two or more times while only updating $\bar{x}_k$, starting with $\bar{x}_k$ equal to $\hat{x}'_k$. If the filter performs in a reasonable manner this iteration will move $\bar{x}_k$ toward a point x* such that

$$z_k = h(x^*) \qquad 20$$

which is "near" the original point $\hat{x}'_k$ in some sense. The difficulty with linearization about a point that does not satisfy equation (20) can be seen from the following simple example.

Consider a case where the predicted state vector $\hat{x}'_k$ is

$$\hat{x}'_k = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and the measurement equation is of the form:

$$z_k = h_k(\underline{x}_k) + v_k = .1\, x_k + (x_k - y_k)^2 + v_k \qquad 21$$

$$p(v_k) = N(v_k,\ .1) \qquad 22$$

For this measurement function:

$$p(z_k | x_k,\ y_k) = N(z_k - .1 x_k - (x_k - y_k)^2,\ .1) \qquad 23$$

Using the linearization procedure of the extended Kalman filter and linearizing about $\hat{x}'_k$, i.e., (0, 0) we have:

$$h_k^L(x_k,\ y_k) = h_k(0,\ 0) + \frac{\partial h_k(0,\ 0)}{\partial x_k} + \frac{\partial h_k(0,\ 0)}{\partial y_k}\, y_k$$

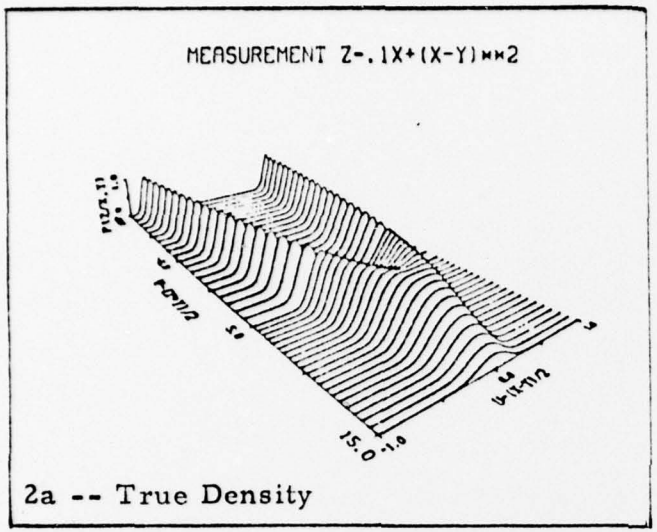
or

$$h_k^L(x_k,\ y_k) = 0.1\, y_k \qquad 24$$

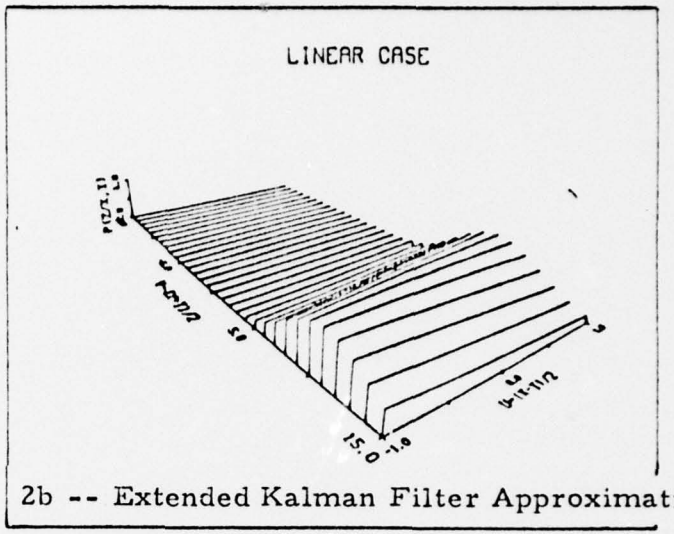
and using this to replace $h_k$ in the measurement density the density used by the Kalman filter to represent equation (23) is:

$$p_{KAL}(z_k | x_k,\ y_k) = N(z_k - .1 x_k,\ .1) \qquad 25$$

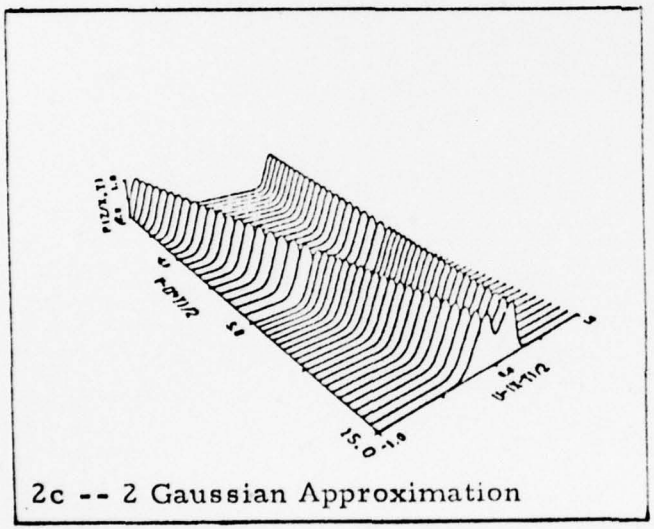
The true function and this usual approximation to it are shown in Figures 2a and 2b, respectively for the case when $z_k$ is taken equal to one. The fact that the nonlinearity is quite large with respect to the linear



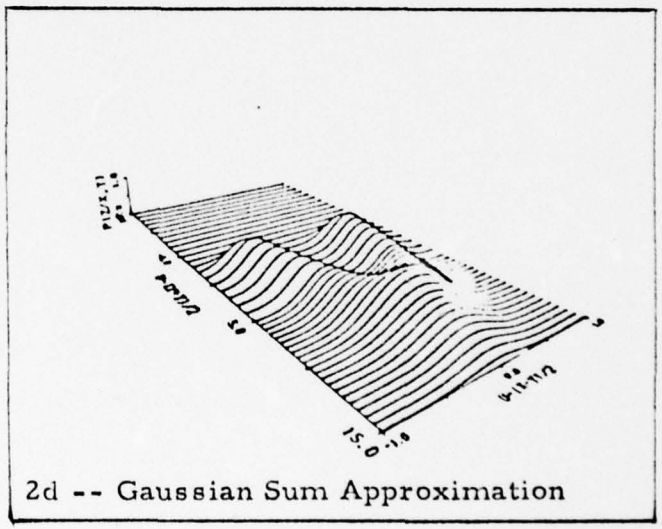

2a -- True Density




2b -- Extended Kalman Filter Approximation



2c -- 2 Gaussian Approximation



2d -- Gaussian Sum Approximation

Figure 2. Measurement Density Function and Approximation

term is born out by the fact that the linearization about the value of states predicted before the $k^{th}$ measurement leads to a very poor measurement density function approximation and thus to a very poor a posteriori density function approximation.

In many cases it is preferable if the function $h_k$ can be linearized about a point when:

$$z_k - h_k(x_k, y_k) = 0 \qquad 26$$

This region is described by the equation:

$$z_k - .1x_k - (x_k - y_k)^2 = 0 \qquad 27$$

with $z_k = 1$.

Which is essentially a U shaped region extending to infinity in two directions. This region is shown for a limited space in Figure 2a as the U shaped ridge on the true measurement density.

Consider the iteration procedure which picks out values closest to the previous expected value or mean of the state $\hat{x}'_k$ and satisfying (27). Note that all values of $x_k$ and $y_k$ such that:

$$y_k(x_k) = y_k \pm \sqrt{z_k - .1x_k} \qquad 28$$

$$x_k(y_k) = y_k - .05 \pm \sqrt{z_k + .0025 - .05\, y_k} \qquad 29$$

satisfy equation (27). Using this and replacing the value appearing on the right hand side of these equations by the previous mean four

points are found:

| i | $\hat{x}_{k_i}$ | $\hat{y}_{k_i}$ |
|---|---|---|
| 1 | 0 | $\sqrt{z_k}$ |
| 2 | 0 | $\sqrt{z_k}$ |
| 3 | $-.05+\sqrt{z_k+.0025}$ | 0 |
| 4 | $-.05-\sqrt{z_k+.0025}$ | 0 |

Figure 3. Points Satisfying Equation (27)

In linearizing the nonlinearity about each of these four points the four resulting functions reduce to two giving the two approximation to $p(z_k|\underline{x}_k)$ shown in Figure 2c. It is seen that this is a considerably better approximation to the true function than that of Figure 2b, and that it might be improved still more by an increase of the predicted variance of each of the Gaussians. It should also be noted that iterating the linearization to its limit point will lead to the right or left hand ridge of Figure 2c, which is considerably better than the non-iterated case but which ignores the second ridge completely.

An alternate approach would be to require that the linearization occur at the point nearest $\hat{x}'_k$ satisfying equation (26). This would only be acceptable if the measurement data was perceived to be more accurate than previous state information.

## 5. GAUSSIAN SUM APPROACH

When the linearization approximations discussed in the last section are not satisfactory, one must return to the general Bayesian recursion relations of section 3. It is generally not possible to solve these equations analytically and some numerical or approximation technique must be used. These include the point mass method of Bucy and Senne [7], spline approximations by Jan and Figueirado [8] and the Gaussian sum methods by Alspach and Sorenson [2, 5]. A good summary of these techniques is given in reference [9].

The Gaussian sum implementation is used here. However, the example is considered more important than the methods, so that only a brief outline of the methods is given. For a more complete description of the approach the reader is referred to the references.

The basic approach for this problem is to approximate the measurement density $p(z_k|x_k)$ by a weighted sum of Gaussian or a Gaussian sum.

$$p(z_k|x_k) = N(z_k - h(x_k),\ R_k) \tag{30}$$

$$p_A(z_k|x_k) = \sum_{i=1}^{M} \alpha_i\, N(z_k - H_{ki}\, x_k,\ R_k) \tag{31}$$

$$\sum_{i=1}^{N} \alpha_i = 1 \qquad \alpha_i \geq 0 \tag{32}$$

For the example in section 4, the two ridges in Figure 2c could be a two term Gaussian sum approximation to the function of Figure 2a. A more detailed approximation would be that of Figure 2d where a 30 term Gaussian sum approximation to the function of Figure 2a is used.

Given a Gaussian or a Gaussian sum expression for $p(\underline{x}_k | \underline{z}^{k-1})$ in equation (8) and the Gaussian sum approximation to $p(z_k | \underline{x}_k)$ from equation (31), one obtains an analytical Gaussian sum expression for $p(x_k | z^k)$ from equation (8). Equations (9) and (10) can also be solved analytically given a Gaussian sum expression for $p(x_{k+1} | z^k)$. This can be repeated to give a complete solution for the a posteriori density function based on any measurement realization.

The only reason this approach does not lead to a true reproducing density is that the number of terms in the Gaussian sum for $p(x_k | z^k)$ is larger than the number in $p(x_{k-1} | z^{k-1})$ if no other approximations are introduced. This is discussed at length in the reference where two additional approximations are introduced and analyzed. The first is that if any term in the Gaussian sum has a weighting $\alpha_i$ of less than $\epsilon$ it is dropped. The second approximation is that if two terms in the sum become similar in an $L_1$ sense they are combined and their weighting coefficients ($\alpha_i$, $\alpha_j$) are combined.

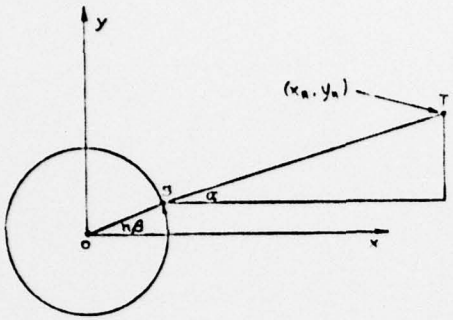
## 6. NUMERICAL RESULTS

The geometry of the situation is repeated in Figure 4a. Then a Gaussian sum approximation is found to approximate the measurement density function $p(z_k | \underline{x}_k)$ which can be written as:

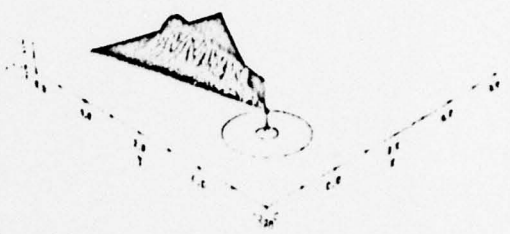
$$p(z_k | \underline{x}_k) = \text{const} \exp [-.5(z_k - h_k(\underline{x}_k))^2 / \alpha_v^2]$$

The true measurement function is shown in Figure 4b where it has been assumed that the measurement density also contains the information that the probability is zero that the target is greater than 6 orbital radii away from the observer. This accounts for the sharp cutoff seen in the figure. Besides this cutoff the major feature of this function that distinguishes it from any possible one Gaussian approximation is the cone shape. This spreads out away from the observer showing that the farther the target is from the observer the larger the possible absolute error in the targets position. For this example the additive measurement noise has a one sigma value of .1 radians or about 6 degrees.

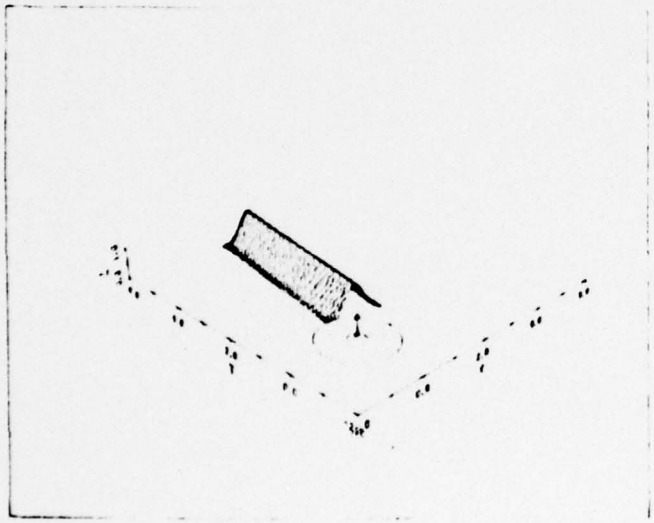
When the basic extended Kalman procedure is used this function, $(h_k)$, is linearized about the previous best estimate for state. When this linearization is performed about any point away from the line of sight of the latest measurement ($z_k = 1$ in this case) very bad results can occur as shown in the example of section 4. These equations were implemented by Bucy and Senne for this example in reference [1] and divergence occurred. In Figure 4c the nonlinearity has been linearized about a point on the line of sight of the measurement and at a distance of 3 units from the observer. This gives the correct value for the variance at the point about which



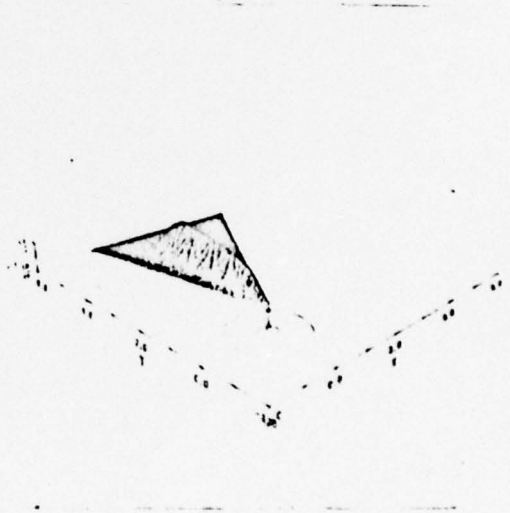

4a



4b



4c



4d

Figure 4. Target/OWN Ship Tracking Geometry

the linearization is made, too small a value for points farther from the observer, and too large a value for points closer to the observer. It can also be seen from Figure 4c that there is no way to incorporate the cutoff data in the one Gaussian approximation in a simple fashion. It is seen that this density function runs to infinity in both directions in the state space plane. Figure 4d shows a 10 Gaussian sum approximation to $p(z_k|\underline{x}_k)$ of Figure 4a which is obtained by the methods of reference [2, 4] and in which no search was required.

This technique is applied to a dynamic example in Figure 5 where the position of the observer is shown by the cross on the unit orbit and the cross on the density function shows the true position of the target. The a priori estimate for the initial state was taken to be:

$$\underline{\hat{x}}_0' = \begin{pmatrix} 2. \\ -.2 \end{pmatrix} \qquad P_0' = \begin{pmatrix} 5. & 0 \\ 0 & 1. \end{pmatrix}$$

while the true value of the initial state (and all subsequent values since there is no plant noise) was taken to be:

$$\underline{x}_k = \begin{pmatrix} 5. \\ .5 \end{pmatrix}$$

The value of $\beta$ on the curve is the value of $\dot{\beta}$ (k-1) where $\dot{\beta}$ is 10 degrees and $\beta_0$ is -90 degrees. The measurement noise here has a one sigma value of .01 radian or about one-half degree. The non-gaussian a posteriori filtering density function is seen to propagate from stage 1 to stage 9 in this figure where a measurement is taken every 10 degrees. There were 6 terms in each Gaussian sum approximation to $p(z_k|\underline{x}_k)$ in this example and the maximum number of

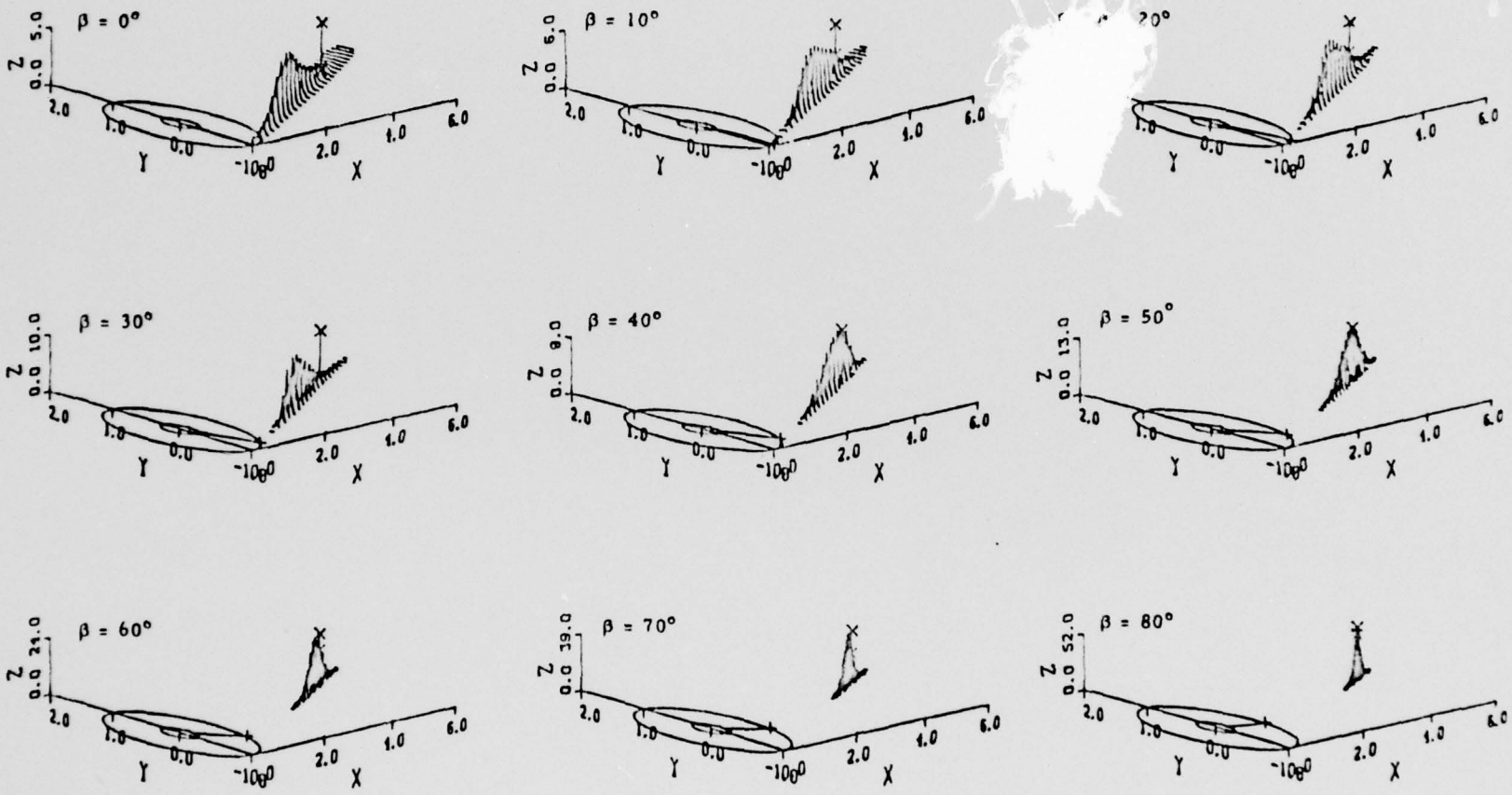



Figure 5. A Posteriori Density Evaluation

terms in any filtering density was 117. The combination and dropping criteria were chosen to be $\delta_1 = \delta_2 = 10^{-4}$.

Figure 6 shows the error in the estimate of the state from the Gaussian sum filter of Figure 5 and the extended Kalman filter. In addition, a divergence measuring parameter which should have an average value of 2. in a Monte Carlo experiment is shown.

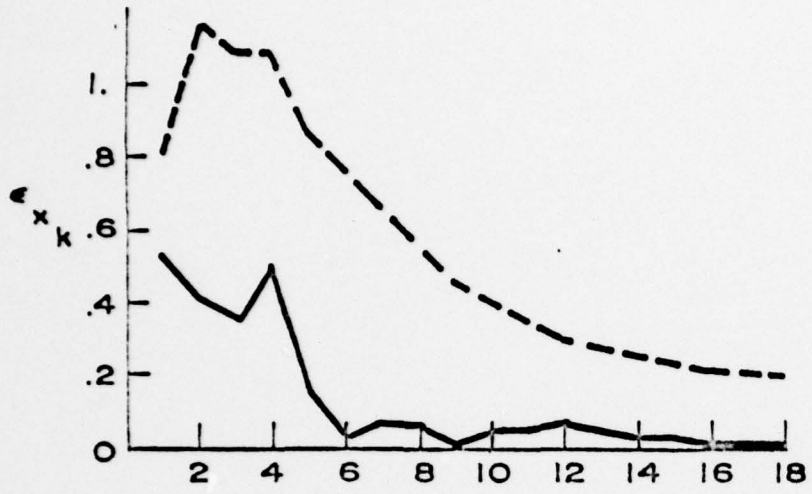
In Figure 7 the measurement one sigma value $\sigma_v$ was increased to .02 radians or about one degree and a plant noise with one sigma value of .1 was added. The statistics of the initial estimate were changed to:

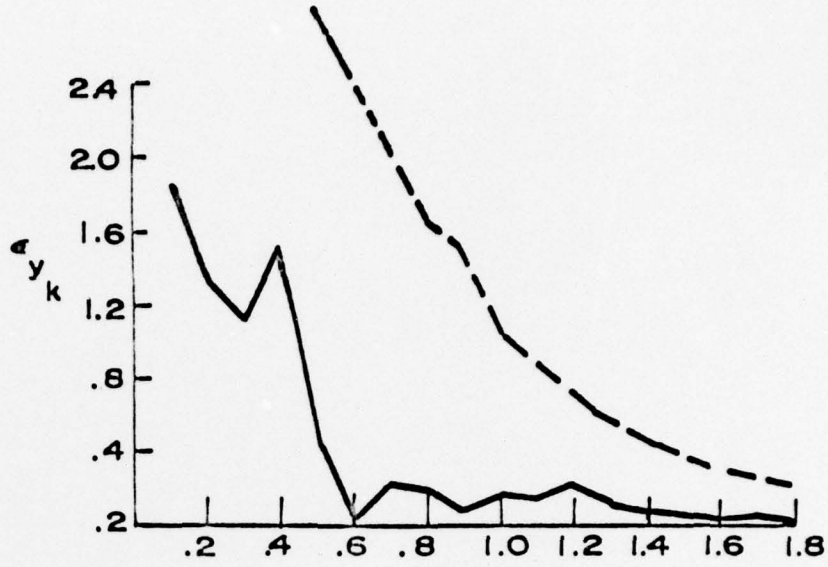
$$\underline{\hat{x}}_0 = \begin{pmatrix} 6. \\ .5 \end{pmatrix} \qquad P'_0 = \begin{pmatrix} 5. & 0 \\ 0 & 2. \end{pmatrix}$$

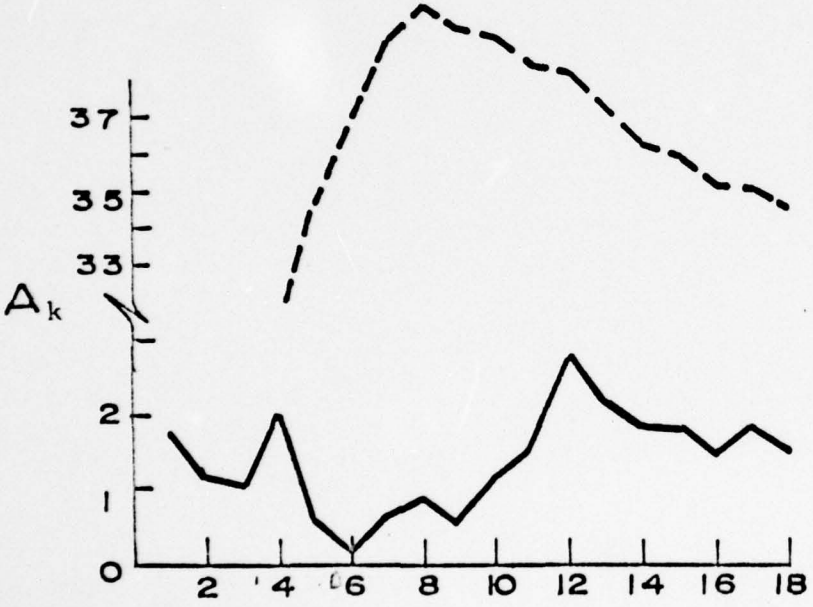
Here the propagation of the distinctly non Gaussian filtering density from stages 1 to 6 is shown.

While these results show the ability of the Gaussian sum approximation to calculate non Gaussian a posteriori densities in a vector case, complete verification of these results have been made by comparison with the considerably more expensive results of Bucy [7] and by a Monte Carlo simulation. Plant noise was added to the system and the initial conditions and a priori statistics were changed to be consistent with the Monte Carlo results presented in [7]. The plant and measurement noise were again white Gaussian sequences with $\sigma^2 = 0.1$, $\beta_0 = 0^o$, $\beta = 1$ rad/stage, $P_o = I$, and

$$\Phi = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix} \quad \hat{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad Q_k = \begin{pmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{pmatrix}$$




$$\underline{\epsilon}_k = \begin{pmatrix} \epsilon_{x_k} \\ \epsilon_{y_k} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} - \begin{pmatrix} \hat{x}_k \\ \hat{y}_k \end{pmatrix}$$

$$A_k = \underline{\epsilon}_k^T P_k^{-1} \underline{\epsilon}_k$$

KALMAN

G.S.

$x_k = 5.0$

$y_k = 0.5$

Figure 6. Kalman Filter/Gaussian Sum Performance Curves

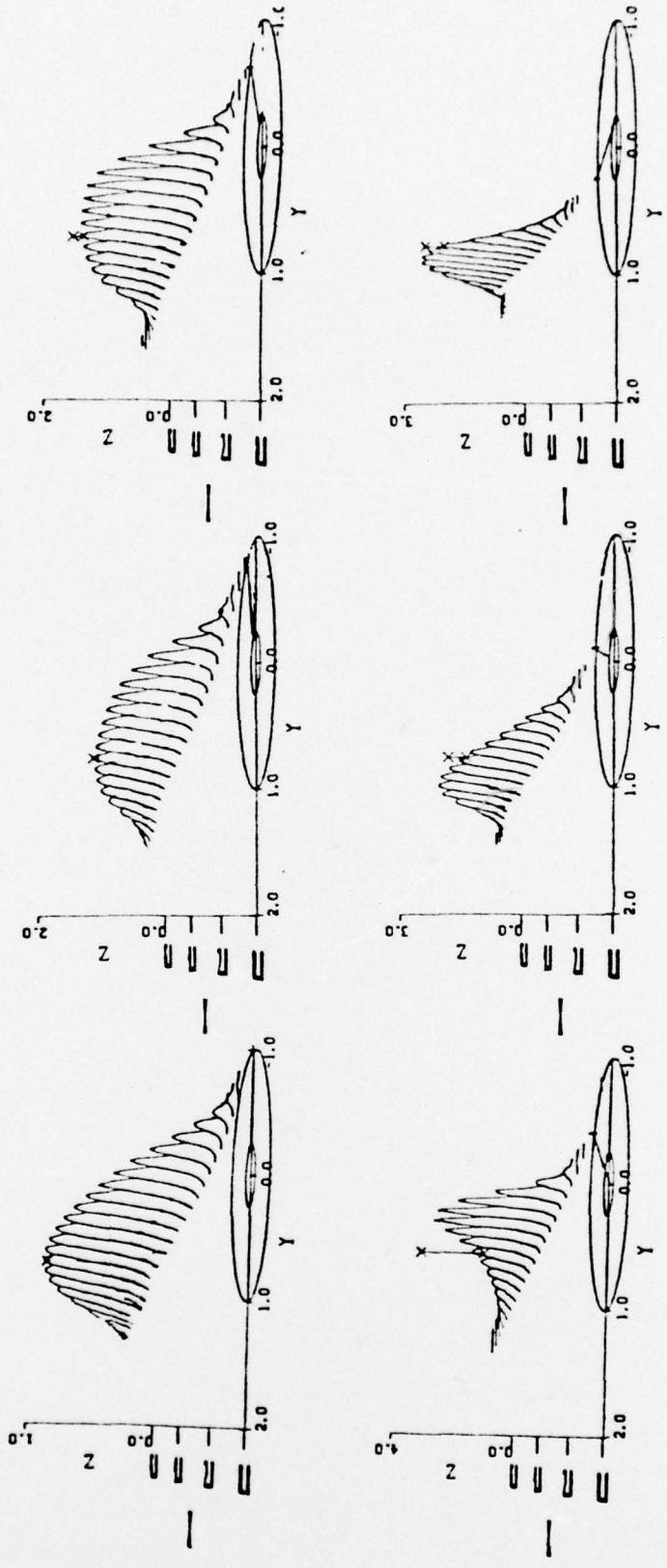



Figure 7. A Posteriori Density Evaluation

A Monte Carlo average of 100 runs was performed filtering the same sample paths for the Gaussian sum and extended Kalman filters. The results are presented in Table 1. With the plant noise added, the increased measurement noise, and the higher rate of rotation of the observer in its orbit, the extended Kalman filter performance with respect to the Gaussian sum filter was greatly improved. Note that this was not true of the more basic extended Kalman filter implemented by Bucy and Senne [7], which was linearized about the latest estimate for state $\hat{x}'_k$ and not about the latest measurement. That version still had severe divergence characteristics. Table 1 shows the results of the filtered estimate for 10 stages. The average error, average covariance, and average divergence parameter are presented. It should be noted from the average covariance that the optimal filter continues to give superior performance even at the later stages. The divergence parameter, which should be such that $A = E(A) = 2$ for an infinite number of runs, is consistently larger for the extended Kalman filter. The large values exhibited by this parameter for some stages result from sample paths for which the covariance of the extended Kalman filter becomes nearly singular. It is worth noting that the results reported here correspond closely with results obtained by Bucy and Senne using the method described in [7]. This has been established in private discussions in which both methods were applied to the same measurement realizations and with all other assumptions identical.

Table 1

MONTE CARLO RESULTS FOR FILTERING DENSITY

| n | Gaussian Sum $e_1$ | Gaussian Sum $e_2$ | Extended Kalman $e_1$ | Extended Kalman $e_2$ | Gaussian Sum $e_1^2$ | Gaussian Sum $e_1e_2$ | Gaussian Sum $e_2^2$ | Extended Kalman $e_1^2$ | Extended Kalman $e_1e_2$ | Extended Kalman $e_2^2$ | Gaussian Sum $A$ | Extended Kalman $A$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | −0.134 | 0.074 | 0.007 | 0.172 | 0.499 | −0.061 | 0.319 | 1.055 | 0.054 | 0.485 | 2.28 | 6.39 |
| 1 | −0.036 | −0.073 | 0.091 | 0.069 | 0.161 | 0.072 | 0.238 | 0.334 | 0.227 | 0.516 | 2.13 | 6.97 |
| 2 | 0.055 | −0.043 | −0.008 | 0.081 | 0.070 | 0.027 | 0.249 | 0.134 | 0.089 | 0.448 | 2.33 | 8.57 |
| 3 | 0.090 | 0.003 | −0.005 | 0.065 | 0.393 | 0.057 | 0.262 | 0.100 | 0.035 | 0.251 | 2.55 | 2.70 |
| 4 | 0.035 | 0.045 | −0.044 | 0.010 | 0.145 | 0.087 | 0.263 | 0.155 | 0.078 | 0.257 | 3.12 | 3.16 |
| 5 | −0.047 | 0.046 | −0.044 | 0.009 | 0.106 | 0.083 | 0.271 | 0.109 | 0.076 | 0.302 | 2.76 | 3.56 |
| 6 | 0.036 | 0.058 | 0.083 | 0.052 | 0.100 | 0.015 | 0.178 | 0.113 | 0.019 | 0.215 | 2.25 | 2.28 |
| 7 | −0.053 | −0.011 | 0.004 | −0.019 | 0.095 | 0.041 | 0.211 | 0.121 | 0.067 | 0.243 | 2.72 | 3.53 |
| 8 | −0.036 | −0.120 | −0.041 | −0.168 | 0.074 | 0.054 | 0.251 | 0.087 | 0.072 | 0.331 | 2.87 | 11.73 |
| 9 | 0.033 | −0.048 | −0.031 | −0.073 | 0.090 | 0.045 | 0.274 | 0.093 | 0.062 | 0.316 | 2.52 | 2.33 |
| 10 | −0.012 | −0.095 | −0.090 | −0.156 | 0.081 | 0.053 | 0.238 | 0.088 | 0.064 | 0.286 | 2.30 | 2.27 |
| 11 | 0.026 | 0.000 | 0.020 | −0.075 | 0.085 | 0.070 | 0.327 | 0.121 | 0.095 | 0.420 | 2.68 | 12.40 |
| 12 | −0.132 | −0.050 | −0.097 | −0.126 | 0.133 | 0.069 | 0.350 | 0.147 | 0.093 | 0.445 | 3.02 | 3.13 |
| 13 | −0.089 | −0.003 | −0.001 | −0.024 | 0.090 | 0.020 | 0.279 | 0.109 | 0.045 | 0.377 | 2.30 | 2.38 |
| 14 | −0.065 | −0.025 | −0.049 | 0.015 | 0.077 | 0.027 | 0.257 | 0.091 | 0.040 | 3.369 | 2.44 | 2.87 |
| 15 | −0.022 | −0.081 | −0.083 | −0.086 | 0.076 | −0.022 | 0.337 | 0.104 | 0.015 | 0.370 | 2.89 | 3.19 |

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